Titles and Abstracts

Keith Ball (University of Warwick)

The probabilistic character of convex domains

Over the last two decades it has become clear that convex domains exhibit properties that we would expect of the joint densities of independent random variables. The developments have been guided by 3 conjectures made 25-30 years ago by Kannan, Lovász and Simonovits, Bourgain and myself.

I will explain how these conjectures fit together and roughly what is known.

Jacob Bedrossian (University of Maryland)

Lower bounds on the top Lyapunov exponent of stochastic systems

We will discuss recent progress on estimating Lyapunov exponents of stochastic models and their application to Eulerian and Lagrangian chaos in stochastic fluid mechanics. For Eulerian chaos, we discuss our recently introduced methods for obtaining strictly positive lower bounds on the top Lyapunov exponent of high-dimensional, stochastic differential equations such as the weakly-damped Lorenz-96 (L96) model or Galerkin truncations of the 2d Navier-Stokes equations. For Lagrangian chaos we discuss our earlier work on positive Lyapunov exponents and almost-sure exponential mixing of scalars by the stochastic Navier-Stokes equations. These hallmarks of chaos have long been observed, however, no mathematical proof had been made for either deterministic or stochastic forcing.
The work on Eulerian chaos combines (A) a new identity connecting the Lyapunov exponents to a Fisher information of the stationary measure of the Markov process tracking tangent directions (the so-called “projective process”); and (B) an L^1-based hypoelliptic regularity estimate to show that this Fisher information is an upper bound on some fractional regularity. For L96 and GNSE, we then further reduce the lower bound to proving the projective process satisfies Hörmander’s condition and introduce a computational framework for checking this condition in certain systems. The work on Lagrangian chaos involves an adaptation of the à la Furstenberg method followed by large-deviation-type estimates to study the two-point dynamics. All of the work discussed is joint with Alex Blumenthal and Sam Punshon-Smith.

Kevin Buzzard (Imperial College London)

The rise of formalism in mathematics
Special lecture on proof formalisation for ordinary mathematicians

Formalism is the art of writing down what you actually mean. Mathematics has a rich history of formalisation: Euclid, Russell–Whitehead and Bourbaki all tried it. This century Avigad, Hales and Gonthier have shown us that there is another way. Now a new generation of young people are formalising algebra, analysis, category theory, combinatorics, geometry, number theory, topology and more at Masters level and beyond, this time using a computer. Lean's mathematics library mathlib contains nearly a million lines of free and open source code corresponding to proofs of over 80,000 theorems such as the fundamental theorem of Galois theory, and it is growing fast. AIs trained on the library have solved IMO problems by themselves. What is happening? This is not about making sure the papers are right. This is not about making a computer program which will print out a one billion line proof of the Birch and Swinnerton-Dyer conjecture using only the axioms of mathematics. This is not about extracting the beauty from a proof and leaving only the directed acyclic graph. This is about developing computer tools which have the potential to help researchers and PhD students in new ways. Much remains to be done. I will give an overview of the area.

Alison Etheridge (University of Oxford)

The motion of hybrid zones (and how to stop them)

Mathematical models play a fundamental role in theoretical population genetics and, in turn, population genetics provides a wealth of mathematical challenges. In this lecture we illustrate this by using a mathematical caricature of the evolution of genetic types in a spatially distributed population to demonstrate the role that the shape of the domain inhabited by a species can play in mediating the interplay between natural selection, spatial structure, and (if time permits) so-called random genetic drift (the randomness due to reproduction in a finite population).

Tadahisa Funaki (University of Tokyo)

Hydrodynamic limit and stochastic PDEs related to interface motion

We consider several types of interacting particle systems at microscopic level, in which particles move performing random walks with or without creation and annihilation depending on the situation. From these systems, via the hydrodynamic space-time scaling limit or its nonlinear fluctuation limit, we derive three different objects at macroscopic level: the motion by mean curvature arising in phase separation phenomena, Stefan free boundary problem describing segregation of species, and coupled KPZ equation which is a system of singular stochastic PDEs. These are all related to the problem of interface motion. The Boltzmann-Gibbs principle plays a fundamental role. We also touch the discrete Schauder estimate.

Craig B. Gentry (TripleBlind, New York)

Homomorphic Encryption

Is it possible to delegate processing of data without giving away access to it? For example, can I query a search engine, and get a useful response, without telling the search engine what I am searching for? Can I send my encrypted financial information to an online tax service, and get back an encrypted completed tax form?

Using an “ordinary” encryption system, it is virtually impossible for someone without the secret decryption key to manipulate the underlying encrypted data in any useful way. However, some encryption systems are "homomorphic" or "malleable". In a homomorphic encryption system, the decryption function is a homomorphism that commutes with operations like addition and multiplication. This homomorphic property allows anyone to manipulate (in a meaningful way) what is encrypted, without knowing the secret key, by operating on ciphertexts.

This talk will survey basic concepts in modern cryptography and complexity theory, including how to prove the security of a system by reducing it to the (assumed) computational infeasibility of well-known mathematical problems, such as factoring large integers. And it will highlight the main ideas behind recent "fully homomorphic encryption" systems, which allow arbitrarily complex functions to be computed on data while it remains encrypted.

Jan S Hesthaven (EPFL)

Structure Preserving Model Order Reduction for Hamiltonian Systems

We discuss the recent developments of projection-based model order reduction (MOR) techniques targeting Hamiltonian problems. Hamilton’s principle completely characterizes many high-dimensional models in mathematical physics, resulting in rich geometric structures, with examples in fluid dynamics, quantum mechanics, optical systems, and epidemiological models. MOR reduces the computational burden associated with the approximation of complex systems by introducing low-dimensional surrogate models, enabling efficient multiquery numerical simulations. However, standard reduction approaches do not guarantee the conservation of the delicate dynamics of Hamiltonian problems, resulting in reduced models plagued by instability or accuracy loss over time.

By approaching the reduction process from the geometric perspective of symplectic manifolds, the resulting reduced models inherit stability and conservation properties of the high-dimensional formulations.

In this presentation we discuss the construction of reduced order models which preserve the symplectic structure through careful construction of the basis itself and extensions to non-linear basis for problems which exhibit low linear reducibility. We shall also briefly discuss the treatment of problems posed on non-canonical form.

The performance of this class of methods is illustrated through a number of computational examples.

Nicholas J. Higham (University of Manchester)

Numerical Stability of Algorithms at Extreme Scale and Low Precisions

As computer architectures evolve and the exascale era approaches, we are solving larger and larger problems. At the same time, much modern hardware provides floating-point arithmetic in half, single, and double precision formats, and to make the most of the hardware we need to exploit the different precisions. How large can we take the dimension n in matrix computations and still obtain solutions of acceptable accuracy? Standard rounding error bounds are proportional to p(n)u, with p growing at least linearly with n. We are at the stage where these rounding error bounds are not able to guarantee any accuracy or stability in the computed results for extreme-scale or low-accuracy computations. We explain how rounding error bounds with much smaller constants can be obtained. The key ideas are to exploit the use of blocked algorithms, which break the data into blocks of size b and lead to a reduction in the error constants by a factor b or more; to take account of architectural features such as extended precision registers and fused multiply – add operations; and to carry out probabilistic rounding error analysis, which provides error constants that are the square roots of those of the worst-case bounds. Combining these different considerations provides new understanding of the limits of what we can compute at extreme scale and low precision in numerical linear algebra.

Hyeonbae Kang (Inha University)

Quantitative analysis of field concentration in presence of closely located inclusions of high contrast

In composites consisting of inclusions and a matrix of different materials, some inclusions are located closely to each other. If the material property of inclusions is of high contrast with that of the matrix, field concentration occurs in the narrow region between closely located inclusions. Understanding the field concentration quantitatively is important in the theory of composites and imaging since it represent stress or field enhancement. Last thirty years or so have witnessed significant progress in analyzing this phenomena of field concentration: optimal estimates and asymptotic characterization capturing the field concentration have been derived in the contexts of the conductivity equation (or anti-plane elasticity), the Lamé system of linear elasticity, the Stokes system. In this talk, we review some of them in a coherent manner.

Sylvie Méléard (École Polytechnique Paris)

Multiscale eco-evolutionary models: from individuals to populations

Motivated by recent biological experiments, we emphasize the effects of small and random populations on long time population dynamics. We will quantify such effects on macroscopic approximations. The individual behaviors are described by the mean of a stochastic measure-valued process. We study different long time asymptotic behaviors depending on the assumptions on mutation size and frequency and on horizontal transmission rate. In some cases, simulations indicate that these models should exhibit surprising asymptotic behaviors such as cyclic behaviors. We explore these behaviors on a simple model where population and time sizes are on a log-scale. Explicit criteria are given to characterize the possible asymptotic behaviors. The impact of the time and size scales on macroscopic approximations is also investigated, leading to Hamilton-Jacobi equations.

Laure Saint-Raymond (IHES)

Dynamics of dilute gases: a statistical approach

The evolution of a gas can be described by different models depending on the observation scale. A natural question, raised by Hilbert in his sixth problem, is whether these models provide consistent predictions. In particular, for dilute gases, it is expected that continuum laws of kinetic theory can be obtained directly from molecular dynamics governed by Newton's fundamental principle.

In the case of hard sphere gases, Lanford showed that the Boltzmann equation emerges as the law of large numbers in the low density limit, at least for very short times. The objective of this talk is to present this limiting process, including recent results on the fluctuations.

David Silver (Deep Mind, London)

Simulation-Based Search

Planning is one of the oldest and most important problems in artificial intelligence. Simulation-based search algorithms such as AlphaZero have achieved superhuman performance in chess and Go and are now used in real-world applications from chemistry to quantum computing. In this talk we present a unified framework for understanding a wide variety of simulation-based search algorithms, including AlphaZero. We conclude by presenting recent results using these algorithms.

Eric Vanden-Eijnden (Courant Institute, New York University)

Enhancing Markov Chain Monte Carlo Sampling Methods with Deep Learning

Sampling high-dimensional probability distributions is a common task in computational chemistry, Bayesian inference, etc. Markov Chain Monte Carlo (MCMC) is the method of choice to perform these calculations, but it is often plagued by slow convergence properties. I will discuss how methods from deep learning (DL) can help enhance the performance of MCMC via a feedback loop in which we simultaneously use DL to learn better samplers based e.g. on generative models, and MCMC to obtain the data for the training of these models. I will illustrate these techniques via several examples, including the sampling of random fields, the calculation of reaction paths in metastable systems and the calculation of free energies and Bayes factors.

Vlad Vicol (Courant Institute, New York University)

Formation and development of singularities for the compressible Euler equations

We consider the compressible Euler equations of fluid dynamics, in multiple space dimensions. In this talk we discuss our program of completely describing the formation and development of stable singularities, from smooth initial conditions. The questions we address are: given smooth initial conditions, precisely how does the first singularity arise? is the mechanism stable? how can one geometrically characterize the preshock (the boundary of the space-time set on which the solution remains smooth)? precisely how does the entropy producing shock wave instantaneously develop from the preshock? does uniqueness hold once the shock has formed? do other singularities instantaneously arise after the preshock? In this level of detail the problem was previously open even in one space dimension. We discuss a sequence of joint works with Steve Shkoller, Tristan Buckmaster, and Theodore Drivas, in which we have developed a multidimensional theory to answer the above questions.