Φ^4_2:

$$\partial_t \Phi = \Delta \Phi - \Phi^3 + \xi_\varepsilon \qquad d=2, \quad x \in \mathbb{T}^2$$

$$+ C_\varepsilon \Phi$$

$$\begin{tikzcd}
\mathcal{F} \arrow[d] & \times (\Pi, \Gamma) \arrow[loop above, "g_\varepsilon"] \arrow[rr] & & \mathcal{D}^\gamma \arrow[d, "\mathcal{R}"] \\
\mathcal{F} \arrow[loop below, "g_\varepsilon"] & \times \; \xi_\varepsilon \arrow[u, "\ell"] \arrow[rr] & & \Phi
\end{tikzcd}$$

$$\Phi = P * \mathbf{1}_{t>0} \left(\xi - \Phi^3 \right) + P \cdot \Phi_0$$

Reg. struc: $T = \operatorname{span}\{ \overset{|k|}{X^k}, \overset{-2^-}{\Xi}, \overset{0^-}{\mathcal{I}(\Xi)}, \overset{0^-}{\mathcal{I}(\Xi)^2}, \underset{0^-}{\mathcal{I}(\Xi)^3} \}$

Canonical lift of smooth ζ_ε : $(\Pi^\varepsilon, \Gamma^\varepsilon)$

$\Pi^\varepsilon_x X^k = (\cdot - x)^k \qquad \Pi^\varepsilon_x \mathcal{I}(\Xi) = K * \zeta_\varepsilon$

$\Pi^\varepsilon_x \Xi = \zeta_\varepsilon \qquad \Pi^\varepsilon_x \mathcal{I}(\Xi)^2 = (K * \zeta_\varepsilon)^2 \qquad \Pi^\varepsilon_x \mathcal{I}(\Xi)^3 = (K * \zeta_\varepsilon)^3$

To deal with discontinuity at $t=0$

define $\mathcal{D}^{\gamma,\eta} \ni f$

$f : (t,x) \mapsto f(t,x) \in T_{<\gamma}$

$$\|f(t,x)\|_\alpha \lesssim \left(1 \wedge \sqrt{|t|}\right)^{(\eta-\alpha)\wedge 0}$$

$$\|f(t,x) - \Gamma_{(t,x),(s,y)} f(s,y)\|_\alpha \lesssim \left(|x-y| + \sqrt{|t-s|}\right)^{\gamma-\alpha} \left(1 \wedge \sqrt{|t|} \wedge \sqrt{|s|}\right)^{\eta-\gamma}$$

$ts > 0$ | ☹ ← restrict to (t,x,s,y) closer to each other than to $\{t=0\}$

Fixed point problem

Taylor lift

$$\Phi = \mathcal{P}\mathbb{1}_{t>0}(\Xi - \Phi^3) + P\Phi_0$$

$$\mathcal{P}F = \mathcal{K}F + (P-K)*\mathcal{R}F$$

$$\Phi = \mathbb{1}_{t>0}\mathcal{I}(\Xi) + \varphi\cdot\mathbb{1}$$

What F.P. problem does $\mathcal{R}\Phi$ solve?

$$\mathcal{R}\Phi = P * \mathbb{1}_{t>0} \cdot \mathcal{R}(\Xi - \Phi^3) + P\Phi_0$$

$$= P * \mathbb{1}_{t>0}(\zeta_\varepsilon - (\mathcal{R}\Phi)^3) + P\Phi_0$$

For canonical lift

Renormalisation: For $C \in \mathbb{R}$, define

$$M_C : T \to T$$

$$M_C X^k = X^k \qquad M_C \mathcal{I}(\Xi) = \mathcal{I}(\Xi)$$

$$M_C \Xi = \Xi \qquad M_C \mathcal{I}(\Xi)^2 = \mathcal{I}(\Xi)^2 - C$$

$$M_C \mathcal{I}(\Xi)^3 = \mathcal{I}(\Xi)^3 - 3C\,\mathcal{I}(\Xi)$$

$$- \tilde{C}$$

$=$ 

Define $\hat{\Pi}^\varepsilon_x = \Pi^\varepsilon_x \cdot M_C$

Renormalised equation

Take $\Phi(x) = \mathcal{I}(\Xi) + \varphi(x)\mathbb{1}$

$$(\hat{\mathcal{R}}\Phi)(x) = \underbrace{(\Pi_x \mathcal{I}(\Xi))(x)}_{v(x)} + \varphi(x)$$

$$(\hat{\mathcal{R}}\Phi)^3 - 3c\hat{\mathcal{R}}\Phi - \tilde{c}$$

$$(\hat{\mathcal{R}}\Phi^3)(x) = (\hat{\Pi}_x \Phi^3(x))(x) = (\Pi_x M_c \Phi^3(x))(x) \quad ''$$

$$M_c\Phi^3(x) = \Phi^3(x) \underbrace{- 3c\varphi(x)\mathbb{1} - 3c\,\mathcal{I}(\Xi)}_{-3c\Phi(x)} - \tilde{c}$$

Want to show that $\exists$ choice of $C = C_\varepsilon$ s.t. $\hat{\Pi}_x^\varepsilon$ converges (in probability) to some model $\hat{\Pi}_x$.

$$\hat{\Pi}_x \bullet = K * \zeta_\varepsilon \qquad \hat{\Pi}_x \vee = (K * \zeta_\varepsilon)^2 - C_\varepsilon$$

$$C_\varepsilon = \mathbb{E}(K * \zeta_\varepsilon)^2(0) \qquad \hat{\Pi}_x \Psi = (K * \zeta_\varepsilon)^3 - 3C_\varepsilon (K * \zeta_\varepsilon)$$

$\xi_\varepsilon = \rho_\varepsilon * \xi \qquad K * \xi_\varepsilon = K_\varepsilon * \xi$

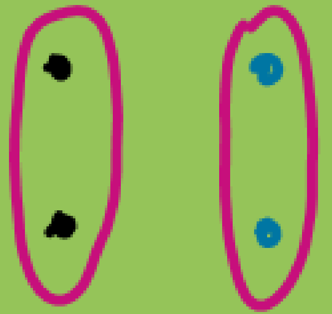
$$\mathbb{E}(K_\varepsilon * \xi)^2(0) = \int K_\varepsilon^2 \, dx\, dt$$

Want to show that $\sim C \cdot \log \varepsilon + \bar{c}_\rho + o(1)$

$$\mathbb{E}\left|\int \psi_x^\lambda \cdot \big(\underbrace{v_\varepsilon^2(y) - C_\varepsilon}_{v_\varepsilon^{\diamond 2}(y)}\big)\, dy\right|^2 = \iint \psi_x^\lambda(y)\psi_x^\lambda(y') \cdot 2 \cdot C_\varepsilon(y-y')^2 \, dy\, dy'$$

$$\lesssim |\log \lambda|^2$$

$$\lesssim \lambda^{-4\kappa}$$



$C_\varepsilon(y-y')$

$= \mathbb{E}\, v_\varepsilon(y)\, v_\varepsilon(y')$