Examples

Φ^4 model $\to$ Stat mech.

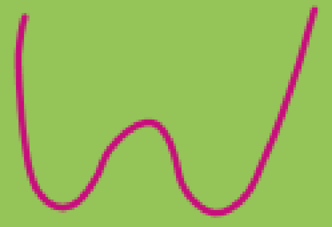
Φ^4 model $\to$ QFT

$$\Phi : \mathbb{R}_+ \times \mathbb{R}^d \to \mathbb{R}$$

$$\mathbb{T}^d$$

$$\partial_t \Phi = \Delta \Phi + \underbrace{C\Phi - \Phi^3}_{-V'(\Phi)} + \xi$$

ξ ← Space-time white noise



$$\mathbb{E}\, \xi(x,t)\,\xi(y,s) = \delta(y-x)\,\delta(t-s)$$

$$\mathbb{E}\, \xi(\varphi)\,\xi(\psi) = \langle \varphi, \psi \rangle_{L^2}$$

Regularity of ξ? $\quad \xi \in \mathcal{C}^\alpha \quad \alpha < 0$?

$r_\alpha \in \mathbb{N} \quad r_\alpha > -\alpha \qquad \mathcal{B}_r = \{ \varphi \in \mathcal{C}^\infty \ \operatorname{supp} \varphi \in B(0,1)$

$\left(\frac{t-s}{\lambda^2} \right. \quad |D^{(k)} \varphi| \leq 1 \ \forall k \leq r \}$

$$\varphi \in \mathcal{B}_r \Rightarrow \varphi_x^\lambda(y) = \lambda^{-d \cdot 2} \varphi\left(\frac{y-x}{\lambda}\right)$$

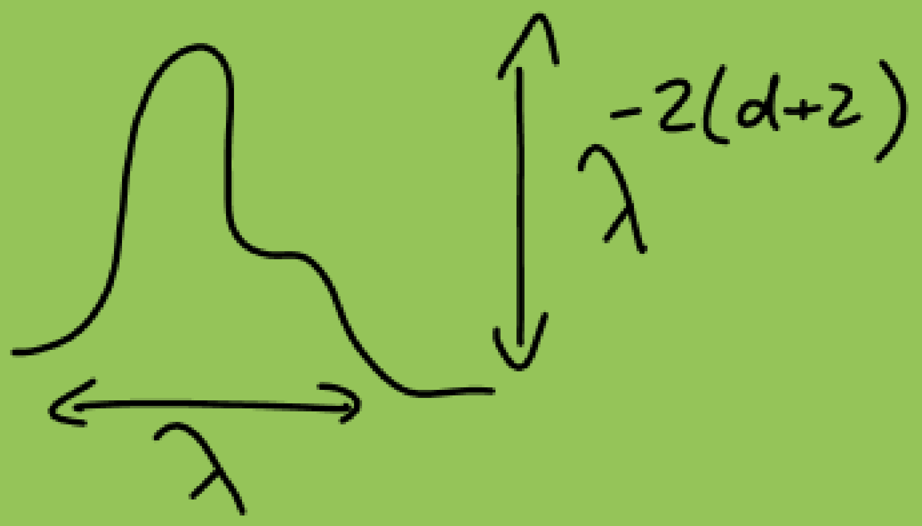
$= (\xi * \varphi_0^\lambda)(x)$

<u>Def</u>: $\xi \in \mathcal{C}^\alpha \ (\alpha < 0)$ iff $\quad |\xi(\varphi_x^\lambda)| \leq C \cdot \lambda^\alpha$

$\forall \lambda \in (0,1], \ \varphi \in \mathcal{B}_{r_\alpha}, \ x \in$ Compact.

$$\left(\mathbb{E}\,\xi(\varphi^\lambda_{(x,t)})^2\right)^{1/2} = \|\varphi^\lambda_{(x,t)}\|_{L^2} \lesssim \lambda^{-\frac{d+2}{2}}$$

$(\varphi^\lambda)^2$



$\Longrightarrow$ STWN has a version with values in $\mathcal{C}^\alpha$ $\forall\, \alpha < -\frac{d+2}{2}$

<u>Thm</u> (Schauder) $\partial_t u = \Delta u + \eta$

$\eta \in \mathcal{C}^\alpha \Rightarrow u \in \mathcal{C}^{\alpha+2}$

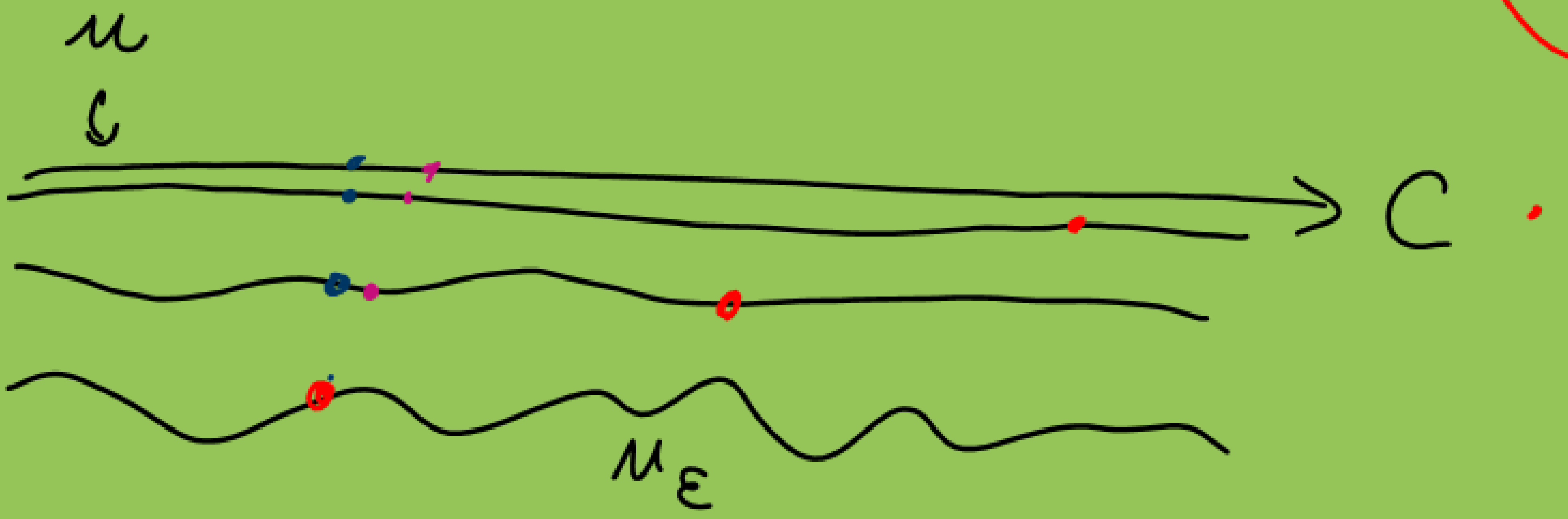
$\eta = \xi \Rightarrow u \in \mathcal{C}^{-\frac{d-2}{2}-}$

$$\partial_t u_\varepsilon = \Delta u_\varepsilon + C_\varepsilon u_\varepsilon - u_\varepsilon^3 + \zeta_\varepsilon \Rightarrow \lim_{\varepsilon \to 0} u_\varepsilon = u$$

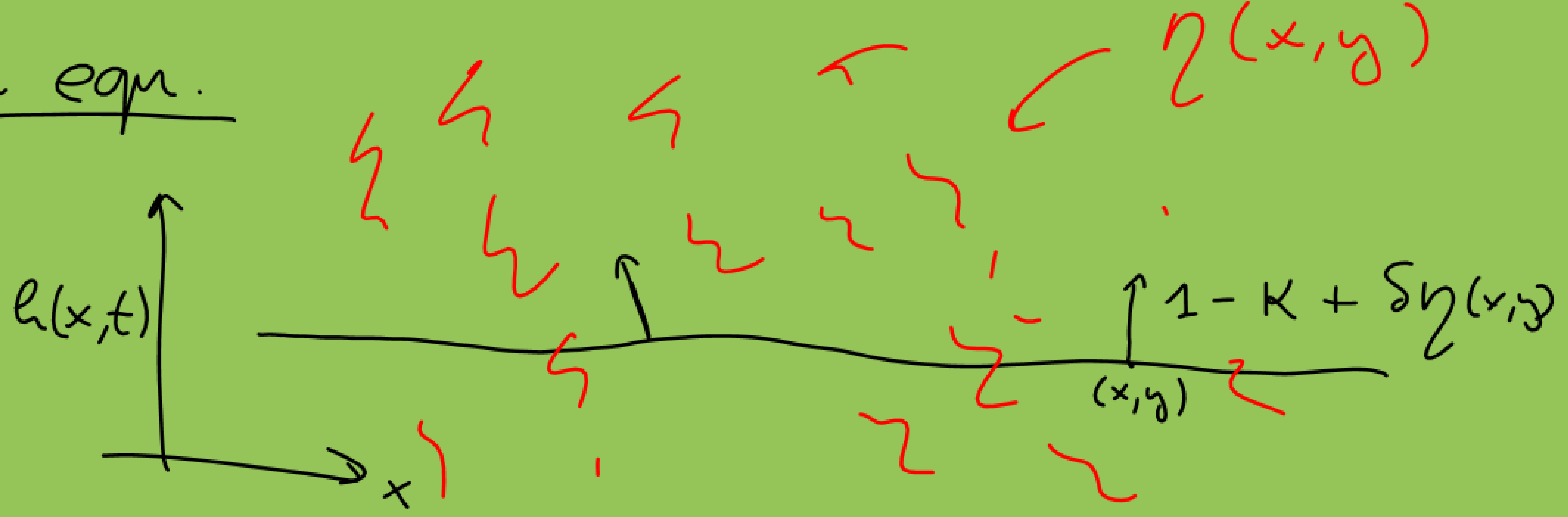
Indep. of ζ_ε

$$d=3 \hookrightarrow C_\varepsilon = \underline{\underline{C}} + \frac{c_1}{\varepsilon} + c_2 \log \varepsilon + c_3$$

Depends on ζ_ε



KPZ eqn.

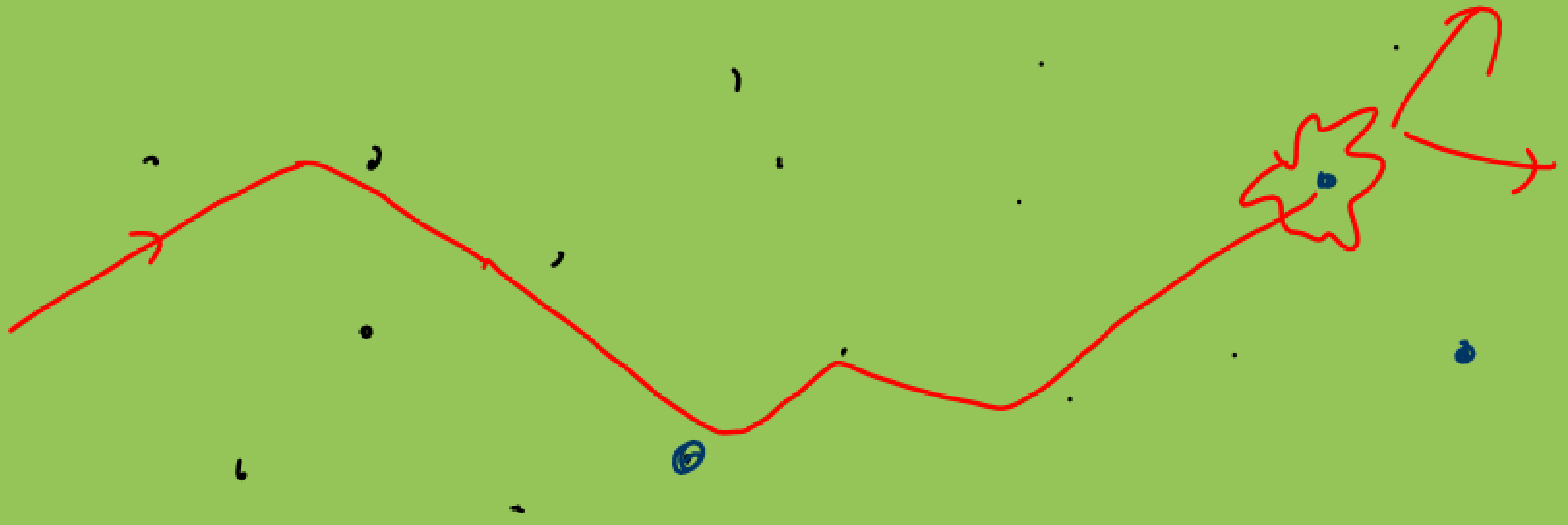


$$\partial_t h = \partial_x^2 h + c(\partial_x h)^2 + \tilde{c} + \xi$$

expect

$u \in C^\alpha \quad \alpha < 1/2$

Continuum Parabolic Anderson



$$\partial_t u = \Delta u + \underbrace{C(x)}_{-\frac{d}{2}} \underbrace{u}_{2-\frac{d}{2}} - C_\varepsilon u \Rightarrow C(x) \text{ white noise in space}$$

$$
\begin{tikzcd}
\mathrm{Eq} \arrow[d] & \times & \mathrm{Model} \arrow[loop above, "\mathcal{R}"] \arrow[r] & \mathcal{D}^{\gamma} \arrow[d] \\
\mathrm{Class\ of\ eq} \arrow[loop left, "\mathcal{R}"] & \times & \mathrm{Noise} \arrow[u, "\mathcal{L}"'] \arrow[r] \arrow[d, Rightarrow] & \mathrm{Sol}. \\
 & & \xi_{\varepsilon} &
\end{tikzcd}
$$

$$
\underset{\mathcal{R}}{\underset{\cap}{M_{\varepsilon}}}\,\mathcal{L}(\xi_{\varepsilon}) \longrightarrow \text{limit}
$$