

$$\partial_t u = \Delta u + Cu - u^3 + \dots \quad x \in \mathbb{T}^3$$

Im d=3 $T = \text{span} \{ X^e, \square, \begin{array}{c} \bullet \\ | \\ \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array} \}$

FP problem: $\begin{array}{c} \bullet \\ | \\ \bullet \end{array}, \begin{array}{c} \bullet \\ | \\ \bullet \end{array}, \begin{array}{c} \bullet \\ | \\ \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array}, \begin{array}{c} \bullet \quad \bullet \\ \diagdown \quad / \\ \bullet \quad \bullet \end{array} \}$

$$U = (\mathcal{K} + \mathcal{R} * \mathcal{R}) \mathbb{1}_{t > 0} (\square - U^3) + P_{u_0}$$

General form of solution to FP:

$$U = i + \mu \cdot \mathbb{1} - \dot{Y} - 3\mu \dot{Y} + \nabla_{\mu} \cdot X$$

$$\begin{aligned} \mathbb{E} - U^3 = & \bullet - \dot{Y} - 3\mu \dot{Y} + 3 \dot{Y} - 3\mu^2 i \\ & + 6\mu \dot{Y} + 9\mu \dot{Y} - 3 \nabla_{\mu} \cdot X \dot{Y} \\ & - \mu^3 \mathbb{1} \end{aligned}$$

Renormalised model

$$M_\varepsilon: T \rightarrow T$$

$$\hat{\Pi}_x^\varepsilon \tau = \Pi_x^\varepsilon M_\varepsilon \tau$$

Would like to take

$$M_\varepsilon \dot{v} = \dot{v} - 3C_\varepsilon \dot{v}$$

$$\hat{\Pi}_x^\varepsilon \Psi = K * \Pi_x^\varepsilon \Psi - (K * \Pi_x^\varepsilon \Psi)_{(x)}$$

$$M_\varepsilon \dot{\Psi} = \dot{\Psi} - 3C_\varepsilon \dot{\Psi}$$

$$\hat{\Pi}_x^\varepsilon \dot{\Psi} = K * \Pi_x^\varepsilon \dot{\Psi} - (\dots)_{(x)}$$

$$M_\varepsilon \Psi = \Psi - 3C_\varepsilon \Psi - C_\varepsilon \dot{\Psi} + 3C_\varepsilon^2 \dot{\Psi} - 3\tilde{C}_\varepsilon$$

If we set $Q: T \rightarrow T$ as the operator that "forgets the red dots"

Then, although $\overline{\Pi}_x^\varepsilon Q\tau \neq \overline{\Pi}_x^\varepsilon \tau$

$$\left(\overline{\Pi}_x^\varepsilon Q\tau \right)(x) = \left(\overline{\Pi}_x^\varepsilon \tau \right)(x) \quad \hat{M}_\varepsilon = QM_\varepsilon$$

Then: $\hat{\mathcal{R}}^\varepsilon F = \mathcal{R}^\varepsilon \hat{M}_\varepsilon F$

$$- \hat{M}_\varepsilon U^3 = -U^3 + 3C_\varepsilon \mathbb{1} + 3C_\varepsilon \mu \mathbb{1}$$

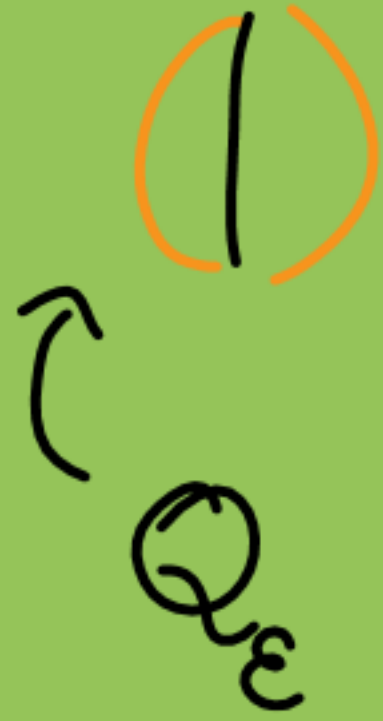
$$- g \tilde{C}_\varepsilon \mathbb{1} - g \tilde{C}_\varepsilon \mu \mathbb{1}$$

$$= -U^3 + (3C_\varepsilon - g \tilde{C}_\varepsilon) U$$

$$C_\varepsilon = \mathbb{E}(\kappa^* \zeta_\varepsilon)^2$$

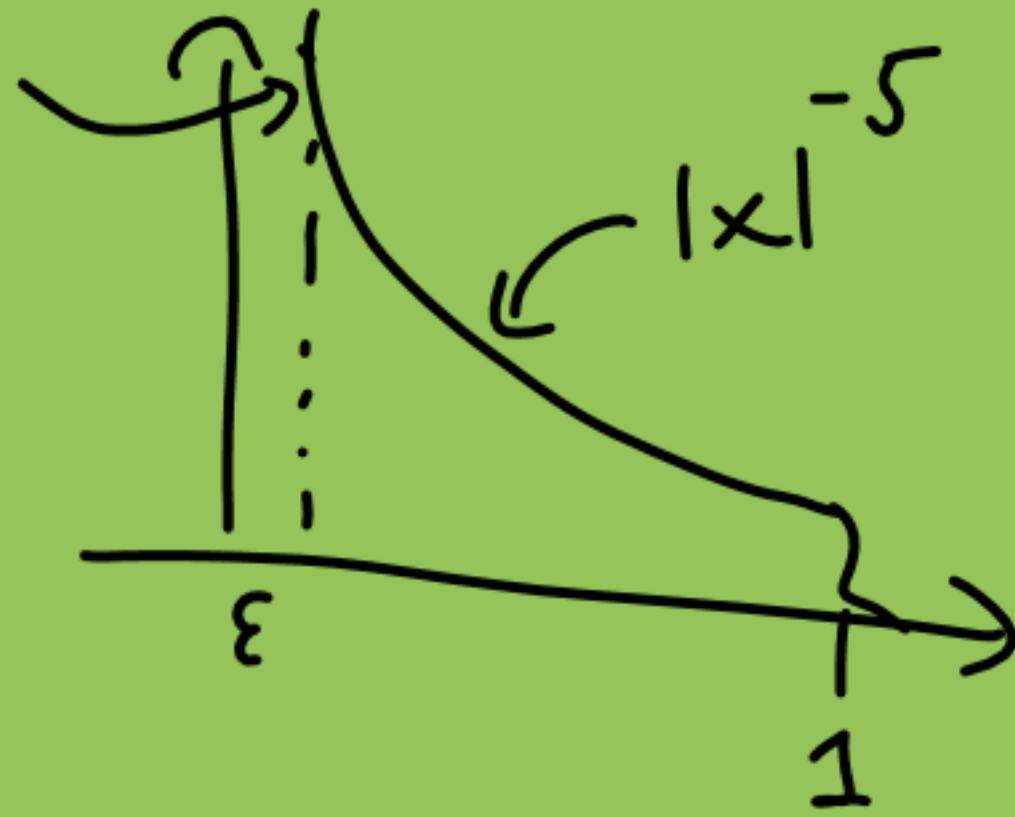
$$\tilde{C}_\varepsilon = \text{Diagram} \sim \log \varepsilon$$





$$\epsilon^{-4} \cdot |\times|^{-1}$$

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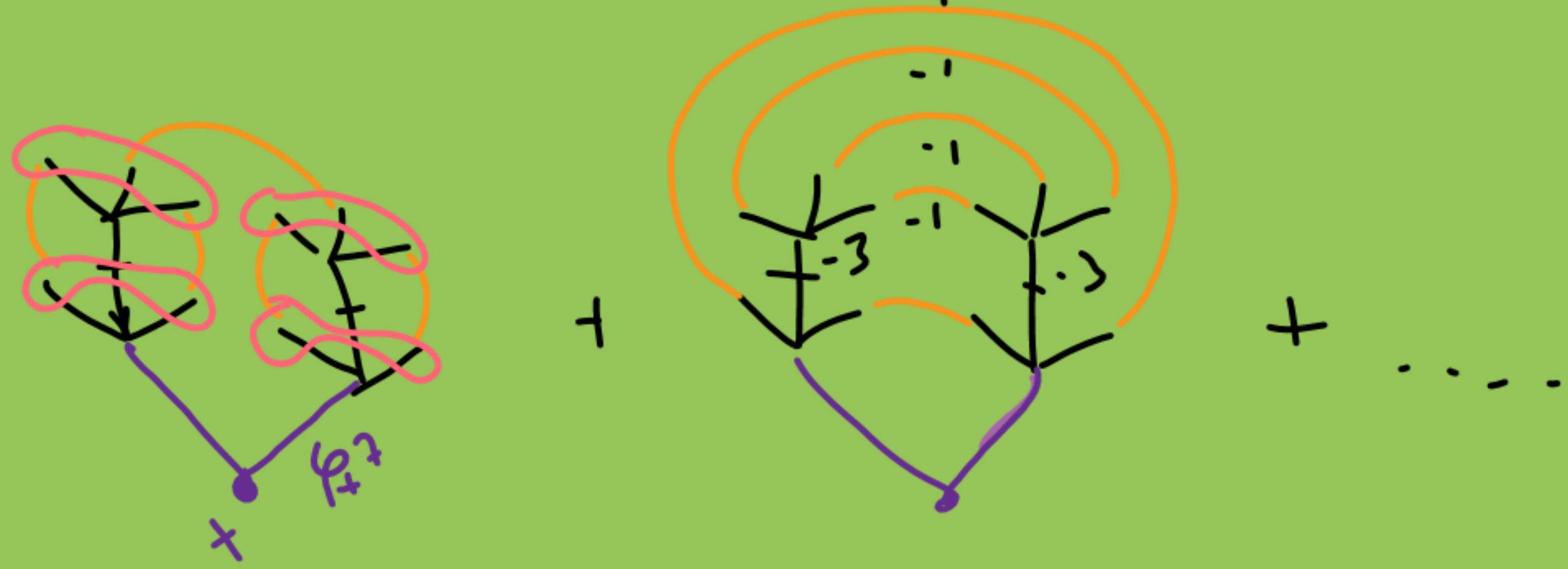
" \mathbb{R}^5 "

2

$$\log \frac{1}{\epsilon} + O(1)$$

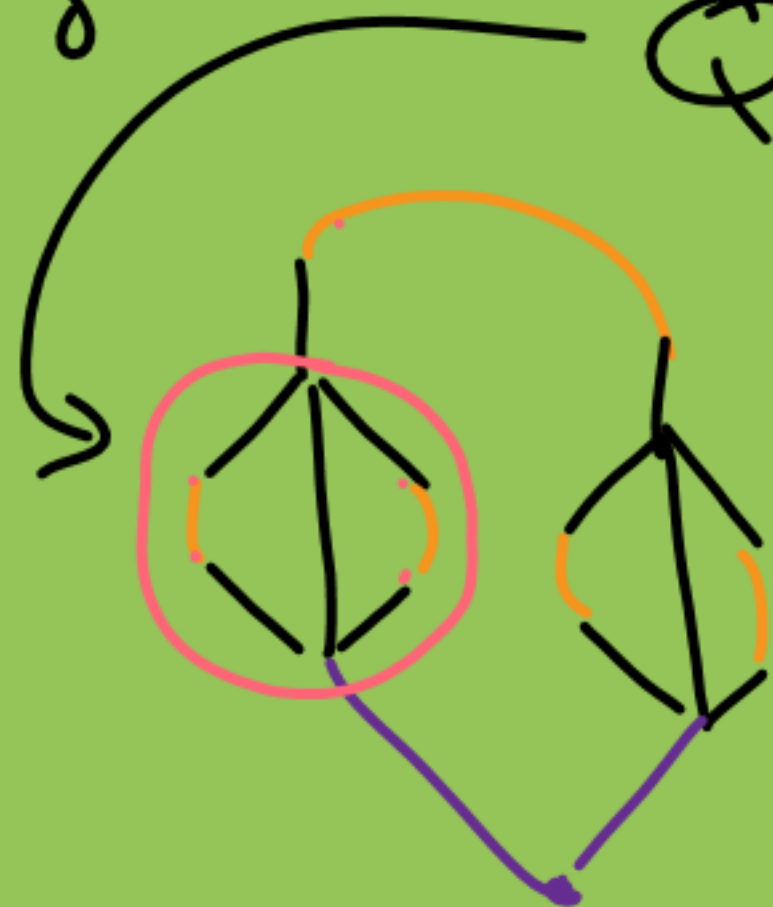
Wanted: $\hat{\Pi}_x^\varepsilon \Downarrow \rightarrow \hat{\Pi}_x \Downarrow$

Really just show $\mathbb{E} \left| \left(\hat{\Pi}_x^\varepsilon \Downarrow \right) \left(\psi_x^\lambda \right) \right|^2 \leq \lambda^{-1}$



Effect of $\tilde{\zeta}_\varepsilon$ is to replace 1st term
by

$$Q_\varepsilon - \delta \cdot \int Q_\varepsilon =: \mathcal{R}Q_\varepsilon$$



$$(\mathcal{R}Q_\varepsilon)(\psi) = \int \psi(x) Q_\varepsilon(x) dx - \psi(0) \cdot \int Q_\varepsilon(x) dx$$

$$= \int \underbrace{(\psi(x) - \psi(0))}_{\leq |x|} Q_\varepsilon(x) dx$$

