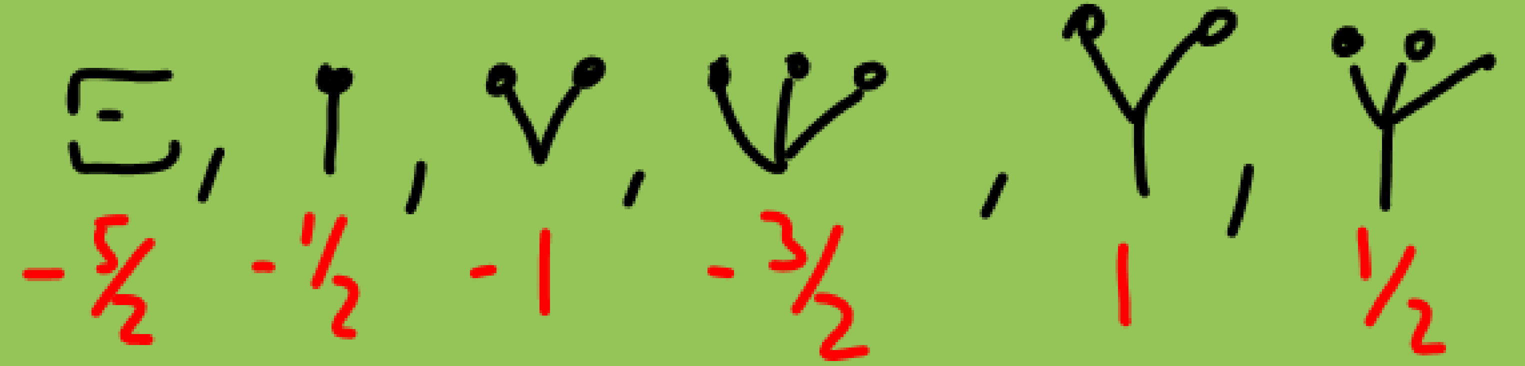
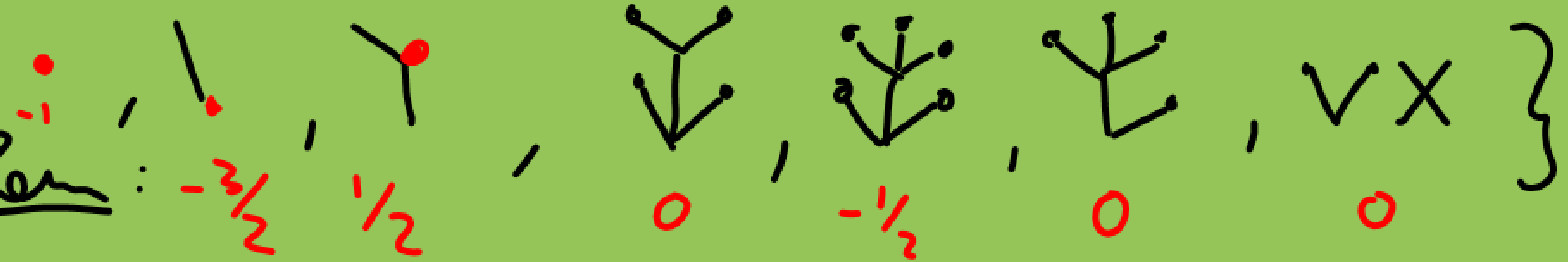
$$\partial_t u = \Delta u + Cu - u^3 + \xi \qquad x \in \mathbb{T}^3$$

In $d=3$

$T = \mathrm{span}\{ X^k,$

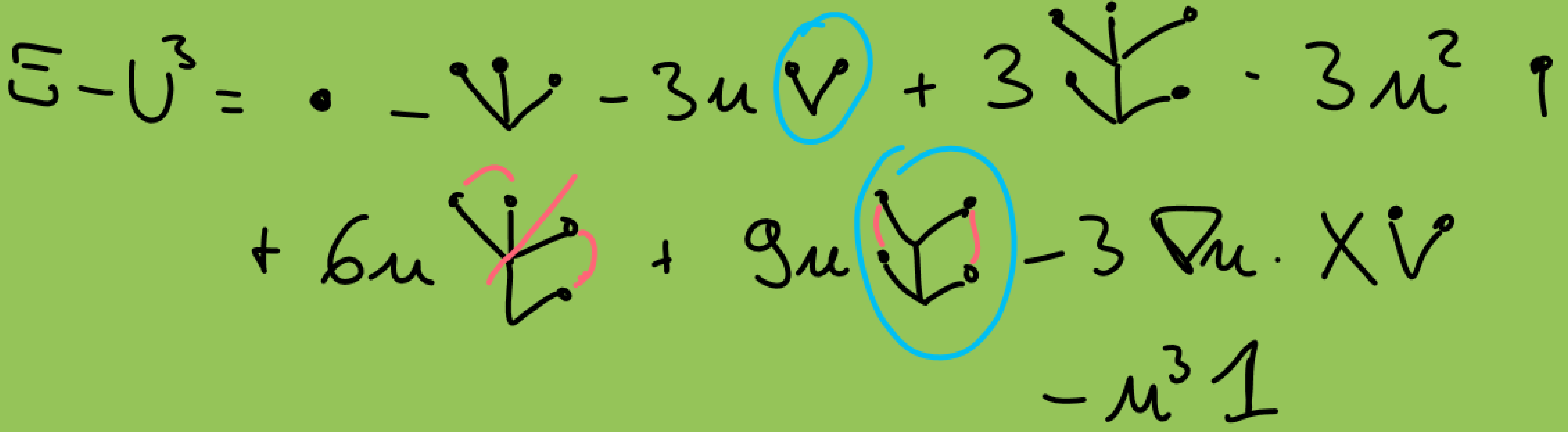


FP problem:



$$U = (\mathcal{K} + R * \mathcal{R})\,\mathbb{1}_{t>0}(\Xi - U^3) + Pu_0$$

General form of solution to FP:

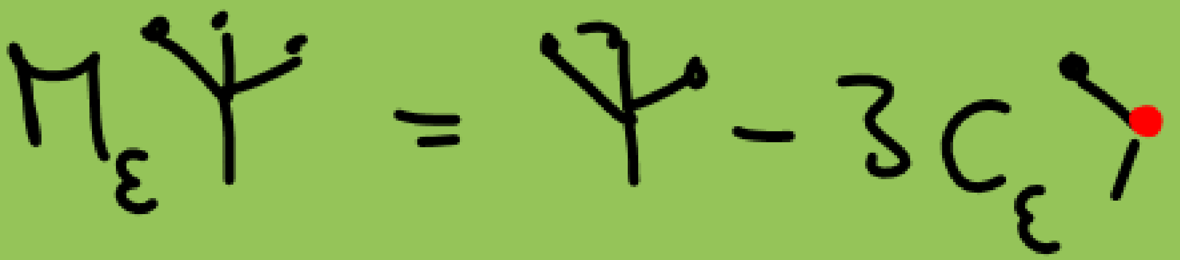


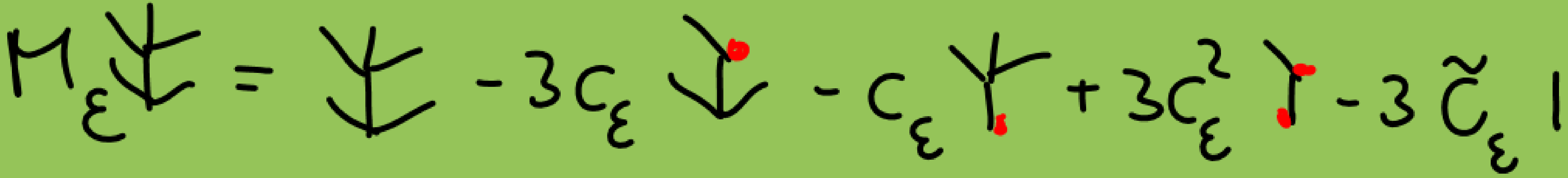
Renormalised model

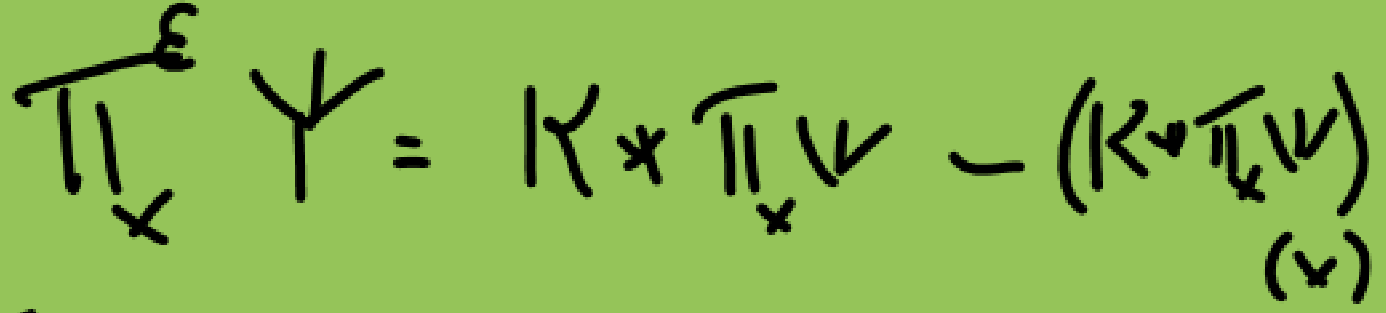
$$M_\varepsilon : T \to T$$

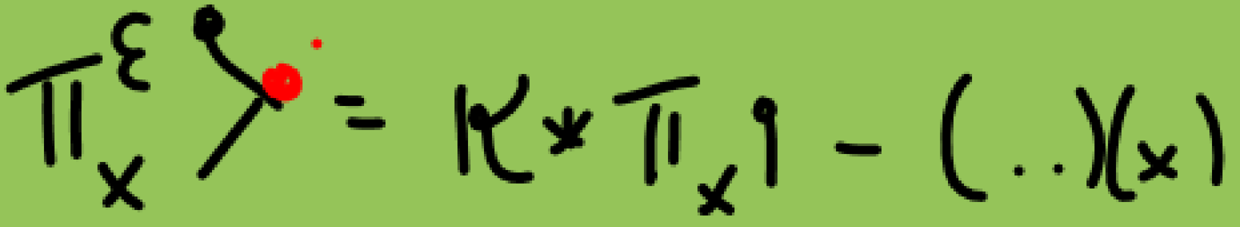
$$\hat{\Pi}^\varepsilon_x \tau = \Pi^\varepsilon_x M_\varepsilon \tau$$

Would like to take









If we set $Q : T \to T$ as the operator that "forgets the red dots"

Then, although $\Pi_x^\varepsilon Q\tau \neq \Pi_x^\varepsilon \tau$

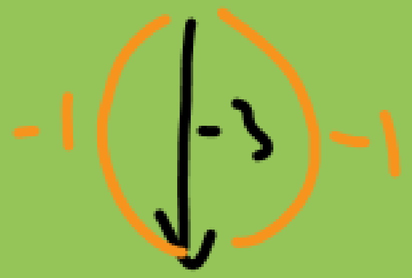
$$(\Pi_x^\varepsilon Q\tau)(x) = (\Pi_x^\varepsilon \tau)(x) \qquad \hat{M}_\varepsilon = Q M_\varepsilon$$

<u>Then</u>: $\hat{\mathcal{R}}^\varepsilon F = \mathcal{R}^\varepsilon \hat{M}_\varepsilon F$

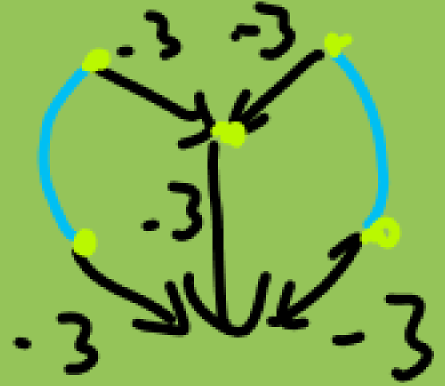
$$-\hat{M}_\varepsilon U^3 = -U^3 + 3C_\varepsilon \dot{|} + 3C_\varepsilon u \mathbb{1}$$

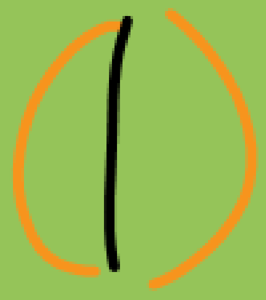
$$- g\tilde{C}_\varepsilon \dot{|} - g\tilde{C}_\varepsilon u \mathbb{1}$$

$$= -U^3 + (3C_\varepsilon - g\tilde{C}_\varepsilon)U$$



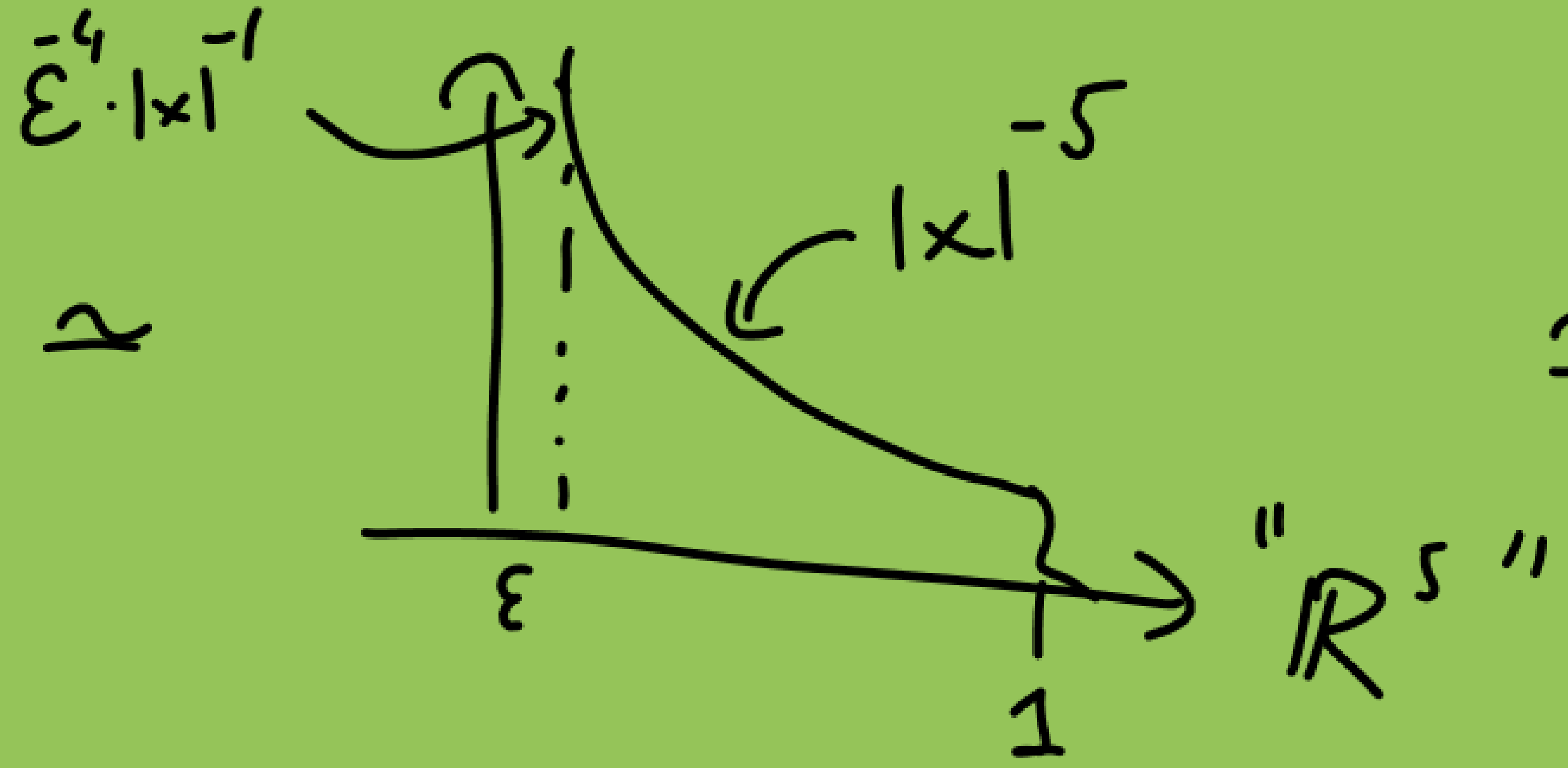
$$C_\varepsilon = \mathbb{E}(K * \xi_\varepsilon)^2$$

$\tilde{C}_\varepsilon =$  $\sim \log \varepsilon$



Q_ε

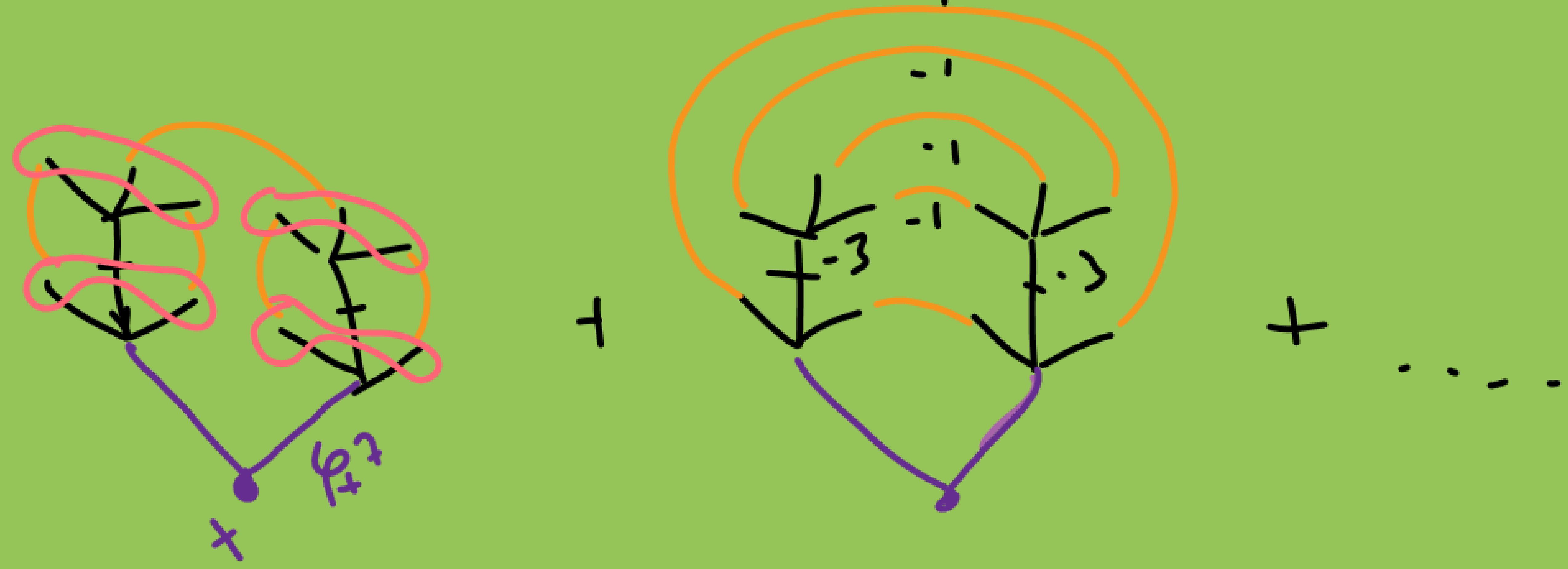
$\simeq$



$\simeq \log 1/\varepsilon + O(1)$

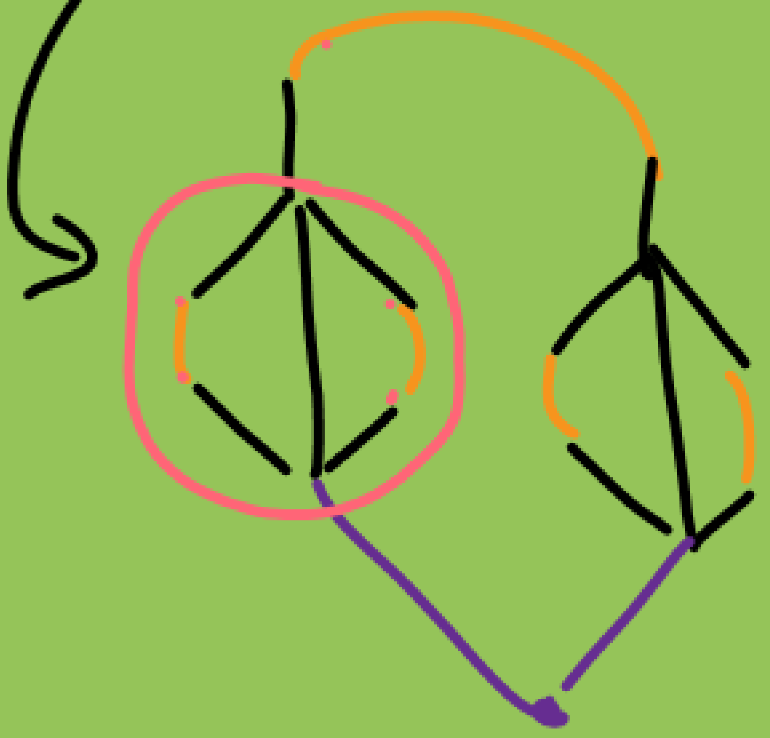
__Wanted__ : $\hat{\Pi}^{\varepsilon}_x \;\longrightarrow\; \hat{\Pi}_x$

Really just show $\mathbb{E}\left|\left(\hat{\Pi}^{\varepsilon}_x\right)\left(\varphi^{\lambda}_x\right)\right|^2 \lesssim \lambda^{-1}$



Effect of $\tilde{C}_c$ is to replace 1st term by

$$Q_\varepsilon - \delta \cdot \int Q_\varepsilon =: \mathcal{R} Q_\varepsilon$$



$$(\mathcal{R} Q_\varepsilon)(\psi) = \int \psi(x) Q_\varepsilon(x)\, dx - \psi(0) \cdot \int Q_\varepsilon(x)\, dx$$

$$= \int \underbrace{(\psi(x) - \psi(0))}_{\lesssim |x|} Q_\varepsilon(x)\, dx$$

$$\mathbb{X}_{s,t} \overset{d}{\neq} \mathbb{X}_{s+r,\,t+r}$$